Analysis and control of the thermal runaway of ceramic slab under microwave heating

Changjun Liu [*] Dongwoo Sheen [†]

October 28, 2007: 01:34

Abstract

Thermal runaway is a special macroscopic phenomenon of the dielectrics during microwave heating, in which there is a big jump of the steady-state temperature while the applied microwave power varies slightly. It hinders engineers in the applications of microwave heating technique in industrial fields. A simulation based on the finite difference time domain (FDTD) method to solve Maxwell's equations coupled with the finite difference (FD) method to solve a heat transfer equation (HTE) is presented in this paper, and the temperature variation in a ceramic slab during microwave heating is obtained. The temperature variation in the ceramic slab during microwave heating is simulated with various ceramic parameters and applied microwave powers so as to analyze the condition under which thermal runaway is introduce. Moreover, a microwave power control method, based on a single temperature threshold and dual applied microwave powers, is presented, which may reduce the microwave heating duration, improve microwave heating efficiency, and thus control thermal runaway. The relation between the final applied microwave power and the temperature threshold in the controlling method is also presented. The analysis method presented in this paper has potential applications in many important fields related with microwave heating techniques.

Key words: Finite difference time domain method (FDTD), heat transfer, temperature, power threshold

1 Introduction

Microwave heating can speed up material processing with many applications in various fields, such as chemical industry, pharmaceutics industry, food industry, mining industry and so

[*]School of Electronics & Information Engineering, Sichuan University, Chengdu 610064, China; E-mail: cjliu@scu.edu.cn

[†]Department of Mathematics, Seoul National University, Seoul 151–747, Korea; *also currently visiting* Department of Mathematics, Purdue University, West Lafayette, IN 47907, USA.; E-mail: sheen@snu.ac.kr

on [1, 2, 3, 4, 5, 6]. During microwave heating, the material characteristics such as the conductivity, permittivity, and heat transfer constant, vary with the temperature rise [7, 8, 9, 10]. It may lead to some special macroscopic thermal phenomenon. When microwave heating is applied to ceramic sintering,[11] mineral grinding,[6] chemistry and pharmaceutics [1, 2, 12], rubber processing, and so on, the local "hot spot" might appear [13]; there may occur a very high temperature rise, which usually leads to the phenomenon of thermal runaway. Such a big temperature jump may result in material degeneration, dielectrics over-burning, or explosion in certain extreme cases. It hinders engineers in the applications of microwave heating in industry.

Thermal runaway is an unstable macroscopic phenomenon during microwave heating, which can be basically described in terms of [14, 15]: a) the steady-state temperature; b) the temperature rise rate. When the microwave power absorbed by a material is equal to the heat loss rate during microwave heating, the system temperature reaches a steady state. If there is a slight variation of the applied microwave power or the parameters of the heated dielectrics, there is an enormous steady-state temperature jump, which describes the steady-state temperature thermal runaway. On the other hand, the temperature rises smoothly until when a certain microwave power is applied, and after that period there occurs a very high temperature rise rate for a very short period of microwave heating. This describes the thermal runaway in terms of the temperature rise rate.

Thermal runaway originates from the dependency of dielectric characteristics on temperature. During microwave heating, if the heat loss rate in a local area is less than the absorbed microwave power, the temperature will rise and lead to the variation of dielectric characteristics. When the variation may function as a positive feedback to increase the microwave power absorption or decrease the heat loss, it will finally introduce a local "hot spot", so called "local thermal runaway". Under a certain condition, the local thermal runaway turns into a global unstable thermal phenomenon. In the process of microwave heating on ceramics, the dependency of either electric conductivity or thermal conductivity on temperature may lead to thermal runaway [9, 14].

In 1991 Kenker et al. presented empirical formula on thermal runaway to study microwave heating on ceramics [16]. Since Kriegsmann analyzed a model of thermal runaway using an asymptotic method in 1992 [14], many researchers have been attracted to discuss mathematical analysis of thermal runaway aspects [15, 17, 18]. In the numerical simulation of thermal runaway, a coupled system of Maxwell's equations and the heat transfer equation (HTE) has to be solved. In 1999, Alpert and Jerby presented a coupling method with two time scales to solve it [19]. Later a finite element method with an adaptive time step scheme has been developed [20]. Some applications of the FDTD method on microwave heating can be found in the literatures [21, 22, 23, 24].

Concerning the control of thermal runaway, Kriegsmann first presented an asymptotical analysis and the control on microwave power to avoid thermal runaway [14]. James and John presented a control method by the initial temperature and the time-varying boundary condition, and performed mathematical analysis [25]. However, it seems that there are many limitations to realize those methods in industry. A practical method is to control microwave power with a temperature feedback system [10, 26]. However, the continuous microwave power adjustment and the detection of the temperature time gradient are complex, difficult and not suitable for industry application

The aim of the present paper is twofold: we first present a new numerical method based on the FDTD method to solve the coupled system of Maxwell's equations and the HTE is presented so as to perform simulation of a ceramic slab under microwave heating; and then we propose a microwave power control method based on a single temperature judgment and dual microwave powers. The steady-state temperature distribution of the ceramic slab depends on the applied microwave power is achieved with various ceramic slab parameters. Meanwhile, the temperature rise in the ceramic slab is obtained by the simulation method. Furthermore, a power controlling method to avoid thermal runway during microwave is discussed. A relation between the final-state microwave power and the monitored temperature threshold is shown, which is valuable in future application of microwave heating. The simulation method presented here enables to simulate more complex microwave heating model, to apply to solving microwave heating apparatus design, and to promote the development of microwave heating research and applications.

The microwave power control method presented in this paper can improve microwave heating efficiency and avoid thermal runaway during microwave heating. Firstly a high microwave power P_H ($P_H > P_{TH}$) is applied, P_{TH} and P_{TL} being the the lower and higher power thresholds of thermal runaway. When the monitored relative temperature on the surface of the ceramic slab is higher than the preset temperature threshold θ_{T}, the applied microwave power is switched to decreases to the final microwave power P_F ($P_{TL} < P_F < P_{TH}$), and maintains the ceramic slab at the desired stable state temperature. Due to high microwave power applied in the initial stage, the temperature rise rate is high in the beginning so as to speed up the process to reach a heat equilibrium state. With several numerical experiments, we conclude that a reasonable value of the preset temperature threshold corresponding to the final applied microwave power will shorten the microwave heating time significantly. Since there are only two microwave power levels as well as one temperature threshold monitor in the proposed method of thermal runway control, the method is convenient, stable and easy to build.

2 Microwave heating model on ceramics

The microwave heating on a ceramic slab in transverse electromagnetic (TEM) mode is facilitated to a model as shown in Figure 1, following [14, 27, 18]). The ceramic slab with length l and relative dielectric constant ε_r is assumed to be exposed in the free space. The microwave with the x- and y-components of the electromagnetic fields, respectively, is assumed to propagates at frequency 2450MHz along the z-direction vertically to the ceramic slab. The electromagnetic fields then satisfy Maxwell's equations:

$$\frac{\partial E_x}{\partial z} = -\mu_r\mu_0\frac{\partial H_y}{\partial t}, \tag{1a}$$

$$\frac{\partial H_y}{\partial z} = \varepsilon_r\varepsilon_0\frac{\partial E_x}{\partial t} + \sigma E_x. \tag{1b}$$

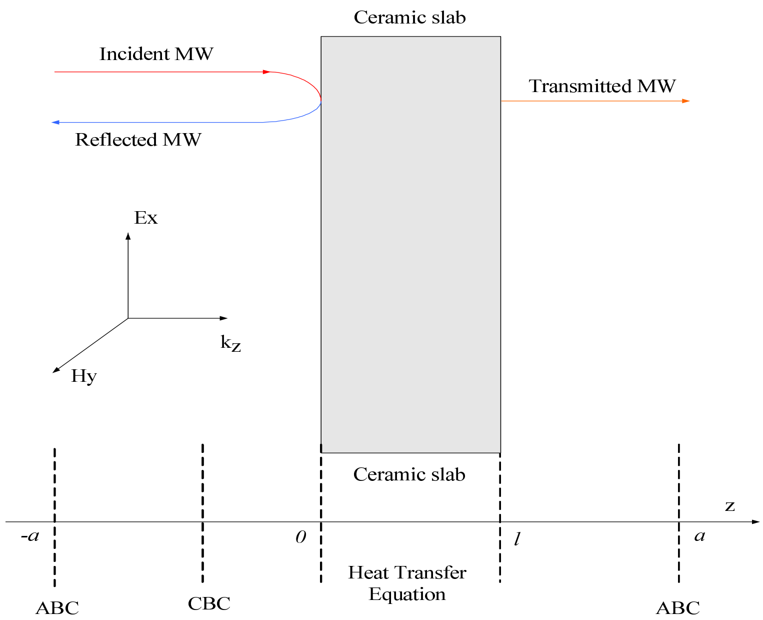


Figure 1.1: Model of microwave heating on a ceramic slab

Since the ceramic slab is a lossy material, it absorbs microwave power and leads to temperature rise. The temperature variation is governed by the HTE:

$$\rho C_p \frac{\partial T(z,t)}{\partial t} = K \frac{\partial^2 T(z,t)}{\partial z^2} + P_d(z,t), \tag{2}$$

where T, ρ, C_p, K and P_d are the temperature, ceramic density, thermal capacitance, thermal conductivity, and microwave power dissipation, respectively. If the polarization loss is not considered, the microwave power dissipation is given by $P_d(z,t) = \frac{1}{2}\sigma(z,t)\left|E_x(z,t)\right|^2$. Assume that the ambient temperature is kept at T_0. The boundary condition of the HTE on the surface of the slab is given by

$$K \frac{\partial T(z,t)}{\partial z} = h\left[T(z,t) - T_0\right] + se\left[T^4(z,t) - T_0^4\right], \tag{3}$$

where h and e are the convective heat and radiation heat constants, respectively, and s=5.67×10$^{-8}kg/s^3/K^4$ denotes the Stefan-Boltzmann constant. The conductivity increases with the rise of temperature, which can be described by an exponential or Arrhenius model [14, 18]:

$$\sigma(T) = \sigma_0 \exp\left(\frac{T - T_0}{T_0}\right), \tag{4}$$

where $\sigma_0 = 1.63 \times 10^{-3}\ S/m$ is the ceramic conductivity at normal temperature $T_0 = 300K$.

3 Simulation method

Since the dielectric characteristics of the ceramics vary with temperature rise under microwave heating, the time-varying medium has to be considered during electromagnetic simulation. A standard FDTD method is applied to solve Maxwell's equations so as to couple the HTE in dealing with time-varying medium [21]. The FDTD simulation system is shown as Figure 1. Outside the ceramic slab the 1^{st}-order Mur absorbing boundary conditions (ABC) are imposed at $z = -a$ and $z = a$. The connection boundary condition (CBC) is then located between the ceramic slab and the left ABC. The time step Δt and the space step Δz in the FDTD method are constrained by the CFL(*Courant-Friedrichs-Lewy*) condition. Here the time step is set as $\Delta t = \frac{c}{2\Delta z}$, with c being the speed of light in vacuum. The finite difference method (FDM) with forward Euler scheme is applied to solve Eq. (2) as follows:

$$\begin{aligned} T^{n+\frac{1}{2}}(k) &= T^{n-\frac{1}{2}}(k) \\ &+ \frac{\Delta t_{HTE}}{\rho C_p}\left[K\frac{T^{n-\frac{1}{2}}(k+1) - 2T^{n-\frac{1}{2}}(k) + T^{n-\frac{1}{2}}(k-1)}{\Delta z^2} + \frac{1}{2}\sigma^{n-\frac{1}{2}}(k)\left|E^n(k)\right|^2\right], \end{aligned} \tag{5}$$

where Δt_{HTE} is the time step, k represents the spatial position $z = k\Delta z$, $\Delta z = l/N_k$, and the superscript denotes $t = n\Delta t_{HTE}$. The stability condition for Eq. (5) is given by $\Delta t_{HTE} < \frac{\rho C_p}{2K}\Delta z^2$. After updating the values of $T^{n+\frac{1}{2}}(k)$ for $k = 1, \cdots, N_k - 1$ using Eq. (5), the value of $T^{n+1/2}(0)$ is updated by the central difference approximation of the spatial derivative of Eq. (3) as follows:

$$T^{n+\frac{1}{2}}(0) = T^{n+\frac{1}{2}}(2) - \frac{2\Delta z}{K}\left[h\left(T^{n+\frac{1}{2}}(1) - T_0\right) + se\left\{\left(T^{n+\frac{1}{2}}(1)\right)^4 - T_0^4\right\}\right]. \tag{6}$$

With respect to microwave at 2450MHz, the time step in the FDTD method is typically at the order 10^{-11}s, while the microwave heating duration normally lasts at the order of 10^2s. If the conventional FDTD method is directly applied, there are at least 10^{13} steps to compute. To speed up the simulation, the time factor α, which is 10^{10}, is applied to transform the time step in the FDTD and HTE into the same order [21, 23, 24]. Figure 1 shows the method to couple Maxwell's equations and the HTE together, in which the electric fields, the magnetic fields, the temperature and the conductivity are updated in turn. The domain to solve the HTE is $[0, l]$, and the domain to apply the FDTD is $[-a, a]$. Based on the model and the coupling method shown in Figs. 1 and 2, the simulation of microwave heating on the ceramic slab can be performed.

4 Simulation results and analysis

The electric field amplitude of the incident microwave is E_0. The dependence of the ceramic permittivity and thermal conductivity on temperature is neglected in the simulation. Other ceramic parameters related with the HTE are obtained from Ref. [19]. Some main parameters in simulation are shown in Table 1.

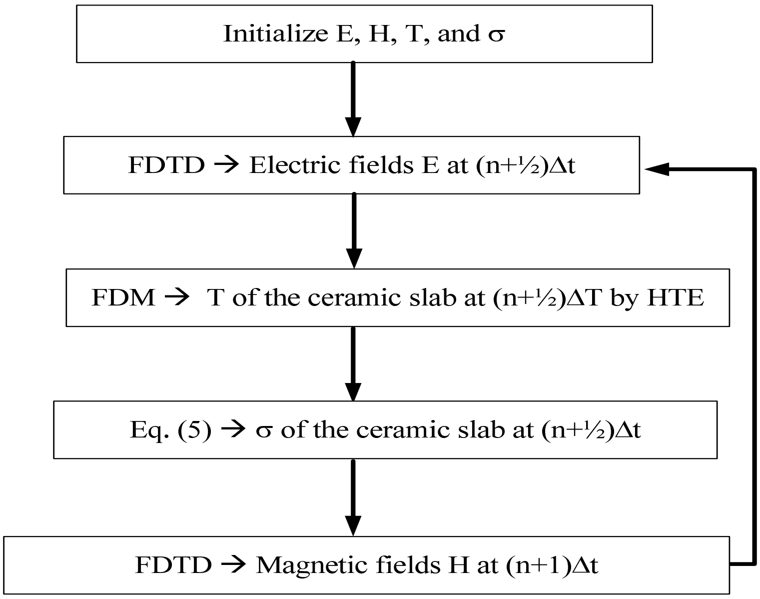


Figure 3.1: Flow chart of the coupling method

Δt^{FDTD} (ps)	Δz (mm)	Δt^{HTE} (ms)	ε_r	K (W/m/K)	e	h (W/m^2/K)
0.504	0.302	5.04	10	10	0.7	170

Table 1: Parameters in FDTD and HTE

4.1 Microwave power threshold of thermal runaway

When the microwave power dissipated of the ceramic slab equals to the rate of heat loss, it reaches the steady-state temperature distribution. The steady-state temperature dependent on microwave power shows the system response of the long duration microwave heating. The thickness of the ceramic slab is 4 cm, and the relative temperature is defined as $\theta = (T - T_0)/T_0$, When the initial temperature of the ceramic slab is of $\theta_0 = 0$ and $\theta_0 = 7$, respectively, the dependence of the steady-state temperature on microwave power is simulated. $\theta_0 = 7$ shows a cooling down procedure, when the absorbed microwave power is less than the heat loss rate. The central temperature θ_C of the ceramic slab is shown in §4.2, where the applied microwave power density is given by $\frac{1}{2}\omega\varepsilon_0\varepsilon_r E_0^2$ (also called microwave power in the paper). The simulated results agree well to those in [18].

If the initial temperature of the ceramic slab is low, the steady-state temperature at the center jumps from the lower branch to the higher branch once the applied microwave power is higher than P_{TH}. If the initial temperature of the ceramic slab is high, the steady-state temperature jumps from the higher branch to the lower branch once the applied microwave power is less than P_{TL}. Thus, there are discontinuous temperature variations dependent on the applied microwave power. The applied microwave powers P_{TL} and P_{TH} are defined as the microwave power thresholds of thermal runaway. At low initial temperature, the slight variation around P_{TH} leads to enormous steady-state temperature change of the ceramic slab, which is the phenomenon of thermal runaway. Until he applied microwave power is below P_{TH}, the steady-state temperature increases smoothly.

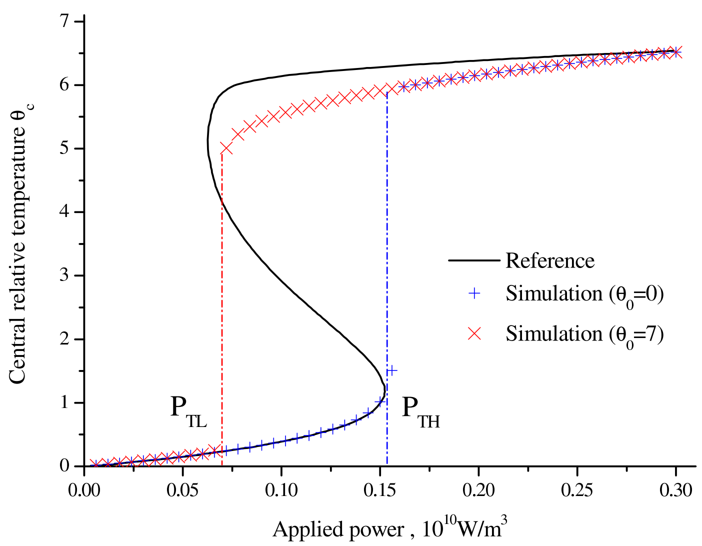


Figure 4.1: S-curve: Steady-state central temperature

4.2 Thermal runaway procedure in time domain

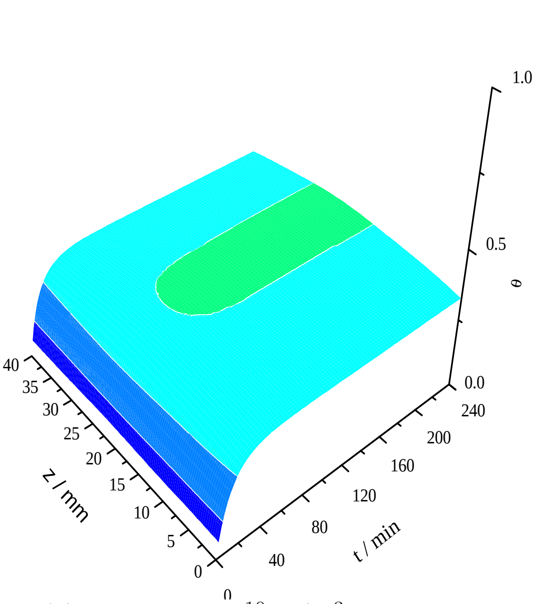
When the initial temperature of the ceramic slab is $\theta_0 = 0$, four microwave powers are applied in order to simulate the temperature rise in the time domain. Figure 4.2(a) shows the temperature rise in the time domain while the applied microwave power is $P_1 = 0.1 \times 10^{10} W/m^3$ ($P_1 << P_{TH}$). The temperature of the ceramic slab increases smoothly while the rate of temperature rise decreases. Finally, at about 80 minutes, the dynamic equilibrium state of temperature reaches, while the highest relative temperature is $\theta_{max} = 0.4$. During the whole microwave heating, the highest temperature is always located at $z = l/2$.

Figure 4.2(b) indicates the temperature rise in the time domain while the applied microwave power is $P_2 = 0.15 \times 10^{10} W/m^3$ ($P_2 \approx P_{TH}$, but P_2 is slightly less than P_{TH}). The temperature of the ceramic slab rises evenly while the rate of temperature rise drops. It takes about 270 minutes to reach a steady state. The highest temperature is again always located at the center of the ceramic slab, and the highest relative temperature is $\theta_{max} = 1.1$. There is no thermal runaway phenomenon in this case.

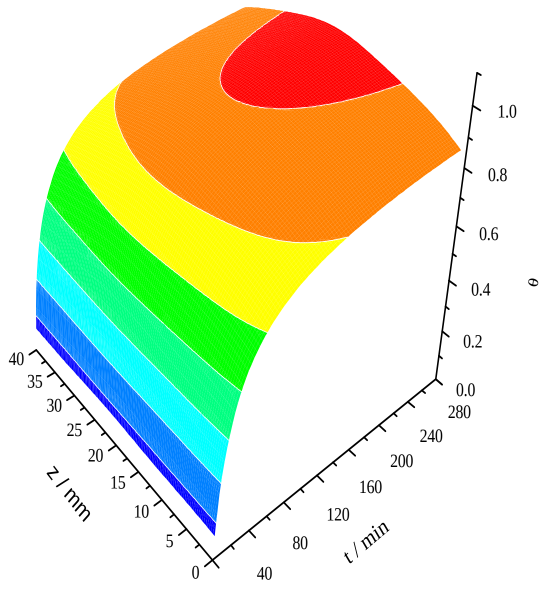
Figure 4.2(c) shows the temperature rise with the applied microwave power $P_3 = 0.16 \times 10^{10} W/m^3$ ($P_3 \approx P_{TH}$, P_3 being slightly greater than P_{TH}). In the beginning stage, the temperature rises smoothly and the highest temperature is located at the center of the ceramic slab. At about 140 minutes of microwave heating, the temperature rises abruptly, and the temperature rise rate goes as high as 326K/min. The phenomenon of thermal runaway appears. At about 200 minutes, the temperature goes to a steady state, while the highest relative temperature is θ_{max}=7.0 at the position near $z = 0$.

The simulation result with the applied microwave power $P_4 = 0.3 \times 10^{10} W/m^3 (P_4 > P_{TH})$ is shown in Figure 4.2(d). The rate of temperature rise speeds up obviously. Although the applied microwave power is only 3 times as high as in that resulted in Figure 4.2(a), the steady-state relative temperature increases about 20 times, which is $\theta_{max} = 8.0$ There is a period of rapid temperature rise and the position of highest temperature switches from $z = 1/2$ to near $z = 0$. At about 20 minutes, the highest temperature rise rate reaches 743K/min. Compared to the case of Figure 4.2(c), the increase of applied microwave power results in much higher temperature rise rate and much earlier thermal runaway; the steady state appears at 40 minutes, while the emergence of thermal runaway advances from 140 minute to about 20 minutes.

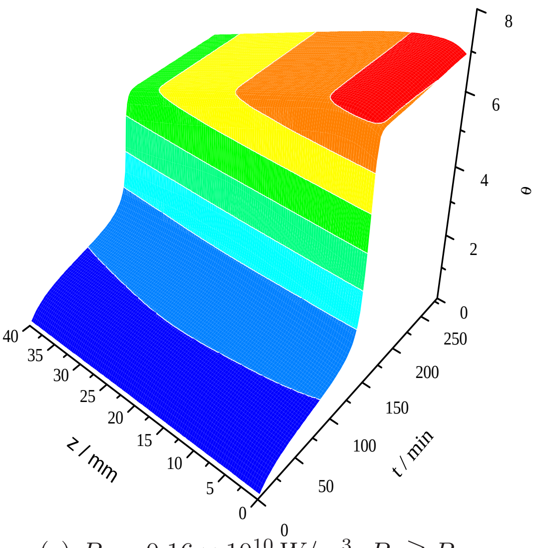
Around the microwave power threshold P_{TH}, a slight power disturbance will lead to a big steady-state temperature jump. Comparing Figures 4.2(b) and (c), the applied microwave power increase only by 6% entails into the highest steady-state relative temperature increase by about 7 times, due to a strong nonlinearity. Furthermore, the closer the applied power is to the power threshold P_{TH}, the longer time is required to reach the steady state. For example, in the cases of Figures 4.2(b) and (c) it takes about 200 minutes to reach the steady state, while it takes only less than 40 minutes to reach the steady state in the cases of Figures 4.2(a) and (d). Once the applied microwave power is higher than the power threshold P_{TH}, there will be a steep temperature rise, as shown in Figures 4.2(c) and (d).



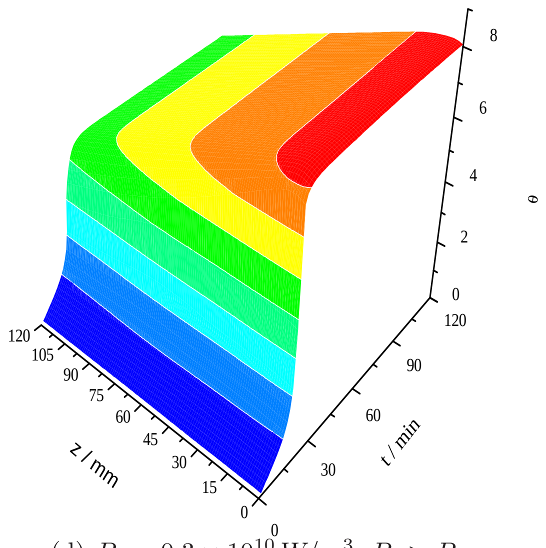
(a) $P_1 = 0.1 \times 10^{10}\, W/m^3\, ; P_1 \ll P_{TH}$



(b) $P_2 = 0.15 \times 10^{10}\, W/m^3\, ; P_2 \lessapprox P_{TH}$



(c) $P_3 = 0.16 \times 10^{10}\, W/m^3\, ; P_3 \gtrapprox P_{TH}$



(d) $P_4 = 0.3 \times 10^{10}\, W/m^3\, ; P_4 > P_{TH}$

Figure 4.2: Time domain temperature rise in the ceramic slab

4.3 Effects on thermal runway from ceramic slab parameters

The variation of ceramic slab leads to the change of the microwave heating effect as well as the microwave power threshold of thermal runaway. The material parameters, such as dielectric constant, heat conductivity, conductivity dependent on temperature, and so on, may change the microwave power threshold of thermal runaway. The geometric parameter (the thickness of the slab) plays a dominant role in the microwave power threshold. When the thickness of the ceramic slab varies, the central temperature of the steady-state relative temperature dependent on the applied microwave power is shown in Figure 4.3. If the thickness of the slab is about 4cm or 6cm, a lower microwave power will lead to the phenomenon of thermal runaway, in which the microwave power threshold corresponding to 6cm is lower than that corresponding to 4cm. If the thickness is about 2.7cm and 4.7cm, much higher microwave power is required to lead to thermal runaway, in which the microwave heating efficiency is low and the relative temperature is lower with the same applied microwave power.

The steady-state temperature is sensitive to the thickness of the ceramic slab at certain condition. For example, with the same applied microwave power, i.e. $P = 0.2 \times 10^{10}\ W/m^3$, a slight thickness variation of 1mm leads to a huge steady-state temperature jump. In the industrial application of microwave heating technique, if there is a slight difference in material under microwave heating, it may result in huge difference in microwave heating effects. If the parameters, such as dielectric constant and the heat conductivity, are changed, similar results may be obtained; a slight variation of one parameter may change the microwave threshold of thermal runaway significantly. Thus, in order to keep the microwave heating system working smoothly, the system should be designed to work in a stable region, in which the temperature is not too sensitive to ceramic parameters.

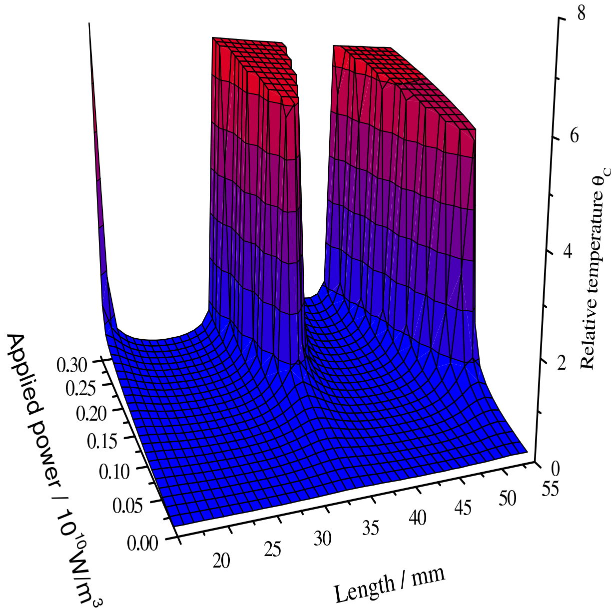


Figure 4.3: Steady-state temperature with the ceramic slab thickness

4.4 The control of thermal runaway

From the simulation of the thermal runaway in time domain it follows that no thermal runaway will be exhibited if the applied microwave power is lower than the microwave power threshold P_{TH}. However, the microwave heating efficiency is very low, especially when the applied microwave power is near P_{TH}. Therefore, in order to increase heating efficiency and to avoid the thermal runaway, a suitable method of microwave power control is required.

A microwave power control method will be discussed, based on a single temperature judgment and dual microwave power, which can improve microwave heating efficiency and avoid thermal runaway during microwave heating. Firstly, a high microwave power P_H ($P_H > P_{TH}$) is applied, recalling that P_{TH} and P_{TL} are the the lower and higher power thresholds of thermal runaway. When the monitored relative temperature on the surface of the ceramic slab is higher than the preset temperature threshold θ_{T}, the applied microwave power will be decreased to the final microwave power P_F ($P_{TL} < P_F < P_{TH}$), and then it will be maintained such that the ceramic slab will be heated at the desired stable state temperature. Since the high microwave power is applied in the initial stage, the temperature rise rate is high in the beginning so as to speed up the process to reach a heat equilibrium state.

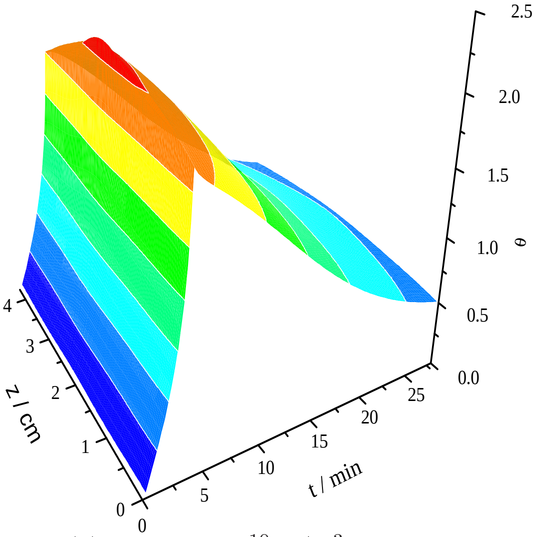
For example, in order to heat a ceramic slab of 4cm thickness, initially the microwave power $P_H = 0.5{\times}10^{10}\ W/m^3$ is applied. When the monitored temperature arrives at $\theta_{\mathrm{T}} = 2.0$ the applied microwave power is switched to the two different cases (a) P_F=0.1×10^{10}W/m^3 and (b) P_F=0.15×10^{10}W/m^3, respectively. During the 25 minutes of microwave heating, the temperature rises are shown in Figures 4.4(a) and (b), respectively. Notice from Figure 4.4(b) that even with the final applied power P_F=0.15×10^{10}W/m^3 that is still lower than the microwave threshold P_{TH}, thermal runaway occurs.

In our further numerical simulation with the preset temperature threshold $\theta_{\mathrm{T}} = 1.0$ and all other parameters unchanged, the temperature rises are shown in Figures 4.4(c) and (d), in which there is no thermal runaway phenomenon. In Figure 4.4(d), observe that the final applied power P_F matches the preset temperature threshold $\theta_{\mathrm{T}} = 1.0$ and that it takes much shorter time to reach the heat equilibrium state. Therefore, a reasonable value of the preset temperature threshold corresponding to the final applied microwave power will shorten the microwave heating time significantly.

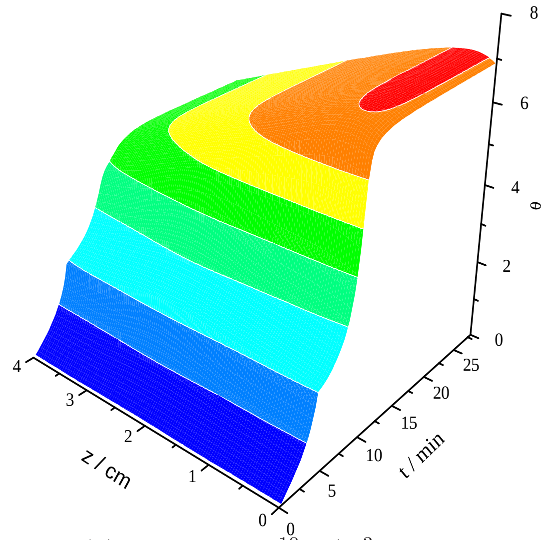
To avoid the phenomenon of thermal runaway, the preset temperature threshold θ_{T}gnd the final applied microwave power P_F are given by

$$\theta_T \leqslant \theta\left(P_{TH}\right) + \frac{\theta\left(P_{TH}\right) - \theta\left(P_{TL}\right)}{P_{TH} - P_{TL}}\left(P_F - P_{TH}\right), \tag{7}$$

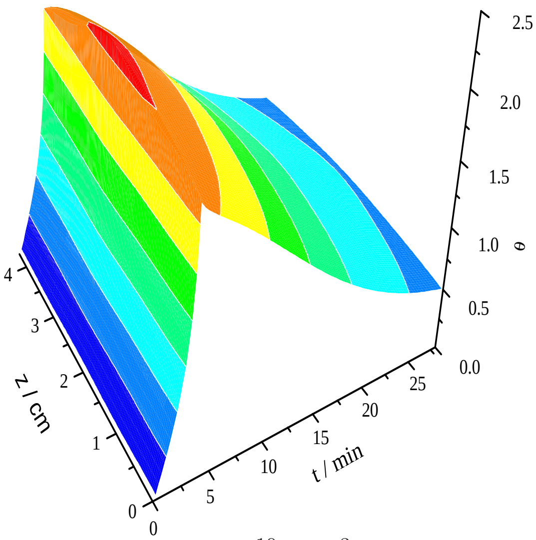
where $\theta(P_{TH})$ and $\theta(P_{TH})$ are the steady-state relative temperature corresponding to microwave thresholds P_{TL} and P_{TH} in the lower and upper branches, respectively (see Figure 4.1). In order to improve the system safety, a reasonable lower temperature threshold θ_T may be adopted. Since there are only two microwave power levels as well as one temperature threshold monitor in the system, the method of thermal runway control is convenient, stable and easy to build.



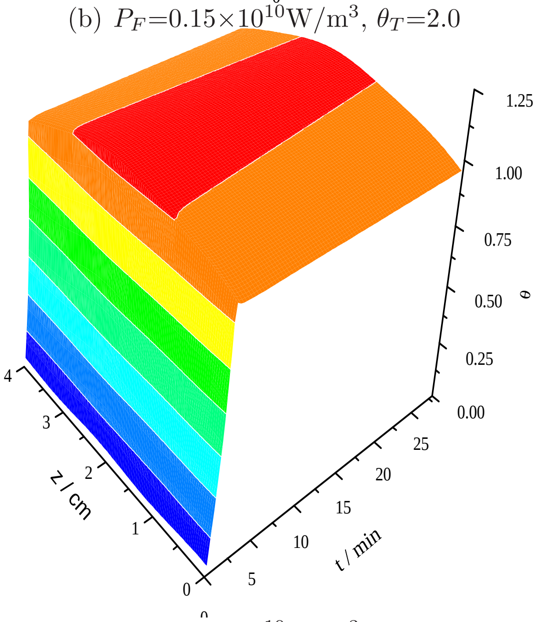
(a) P_F=0.1×10^{10}W/m^3, θ_T = 2.0



(b) P_F=0.15×10^{10}W/m^3, θ_T=2.0



(c) P_F=0.1×10^{10}W/m^3, θ_T=1.0



(d) P_F=0.15×10^{10}W/m^3, θ_T=1.0

Figure 4.4: The control on thermal runaway

5 Conclusion and discussion

The coupling method based on the FDTD method to solve Maxwell's equations and the FD method to solve HTE is applied in this paper to simulate thermal runaway during microwave heating on a ceramic slab. The strong nonlinearity and complexity of the microwave heating process is also achieved. Based on the simulation and analysis, the conditions of thermal runaway are: 1) there is a dependence of material characteristics on temperature; 2) the applied microwave power is greater than a threshold. The microwave power threshold of thermal runaway is not only dependent on material characteristic, but also sensitive to the geometric parameters under certain conditions.

The analysis of the temperature rise in the time domain shows that if the applied microwave power is higher than the microwave power threshold, there is a steep temperature rise during microwave heating; otherwise, the temperature rises smoothly.

Under certain conditions, the microwave power of thermal runaway is sensitive to certain parameters of the material under heating. For example, in the applications of microwave heating to foods, it was found that the temperature rise and distribution were sensitive to the size and shape of the food [5]. From the simulation results shown in this paper, with the same applied microwave power, a slight variation of the ceramic slab thickness may lead to a significant temperature jump of the steady-state temperature. The sensitivity can not be explained by the resonance between the applied microwave and the dielectrics under heating, but by the macroscopic behavior from the bifurcation of the nonlinear model. In the applications of microwave heating, those sensitive regions should be avoided in the design in order to make the microwave heating system more stable.

A method of microwave power control based on single temperature threshold and dual temperature levels is presented in this paper, which can avoid the phenomenon of thermal runaway. The high microwave power is applied at the initial stage to speed up the heating process, while the final microwave power is applied to keep the material in heat equilibrium. Thermal runaway is related with not only the applied microwave power but also to the current material temperature. A formula to estimate the reasonable the final microwave power and the temperature threshold is given.

With the rapid development of microwave techniques, there are many new fields for microwave applications, such as microwave soldering [28], microwave hybrid heating[29], microwave thawing [30], microwave drilling [31], and so on. The Maxwell's equations coupled with the HTE are required to solve to analyze the heating process. The method presented in this paper is applicable in both thermal runaway control and microwave heating applications.

Acknowledgments. The research was supported by NSCF 603010004, NSFC 60411140521, KOSEF F01-2004-000-10263-0, R14-2003-019-01000-0, and KRF-2006-070-C00014.

References

[1] Oliver CK and Doris D. The impact of microwave synthesis on drug discovery. *Nature Reviews Drug Discovery*, 2006, 5(1):51–63.

[2] Adam D. Microwave chemistry: Out of the kitchen. *Nature*, 2003, 421(6923):571–572.

[3] Jerby E, Dikhtyar V, Aktushev O, and Grosglick U. The microwave drill. *Science*, 2002, 298:587–589.

[4] Osepchuk JM. Microwave power applications. *IEEE Trans. MTT*, 2002, 59(3):975–985.

[5] Oliverira MEC and Franca AS. Microwave heating of foodstuffs. *J. Food Engineering*, 2002, 53:347–359.

[6] Walkiewicz JW, Clark AE, and McGill SL. Microwave-assisted grinding. *IEEE Trans. Industry Applications*, 1991, 27(2):239–242.

[7] Wu X and Thomas JR Jr. Control of thermal runaway in microwave resonant cavities. *J. Appl. Phys*, 2002, 92(6):3374–3380.

[8] Nelson SO and Bartley PG Jr. Measuring frequency- and temperature-dependent permittivities of food materials. *IEEE Trans. Instrumentation Measurement*, 2002, 51(4):589–692.

[9] Parris PE and Kenker VM. Thermal runaway in ceramics arising from the temperature dependent of the thermal conductivity. *Phys. Stat. Sol (b)*, 1997, 200:39–47.

[10] Bale GO and Li M. Robust temperature control for microwave heating of ceramics. *IEEE Trans. Industrial Electronics*, 1997, 44(1):124–131.

[11] Chatterjee A, Basak T, and Ayappa KG. Analysis of microwave sintering of ceramics. *AIChE Journal*, 1998, 44(10):2302–2311,

[12] McIntosh RD and Nolan PF. Review of the selection and design of mitigation systems for runaway chemical reactions. *J. Loss Prevention Process Industries*, 2001, 14:27–42.

[13] Zhang Y Zhu S and Marchant TR. A DRBEM model for microwave heating problems. *Appl. Math. Modelling*, 1995, 19:287–297.

[14] Kriegsmann GA. Thermal runaway in microwave heated ceramics - a one-dimensional model. *J. Appl. Phys*, 1992, 71(4):1960–1966.

[15] Hill JM and Marchant TR. Modeling microwave heating. *Appl. Math. Modelling*, 1996, 20(1):3–15.

[16] Kenker VM, Skala L, and Weiser MW t al. Theory of microwave interactions in ceramic materials: the phenomenon of thermal runaway. *J. of Materials Science*, 1991, 26:2483–2489.

[17] Vriezinga CA, Sanchez-Pedreno S, and Grasman J. Thermal runaway in microwave heating - a mathematical analysis. *Applied Mathematical Modelling*, 2002, 26:1029–1038.

[18] Gupta N, Midha V, Balakotaiah V, and Economou DJ. Bifurcation analysis of thermal runaway in microwave heating of ceramics. *J. Electrochemical Society*, 1999, 146(12):4659–4665.

[19] Alpert Y and Jerby E. Coupled thermal-electromagnetic model for microwave heating of temperature-dependent dielectric media. *IEEE Trans. PS*, 1999, 27(2):555–562.

[20] Rabello AA, Silva EJ, and Saldanha RR *et al.* Adaptive time-stepping analysis of nonlinear microwave heating problems. *IEEE Trans. Magnetics*, 2005, 41(5):1584–1587.

[21] Torres FC and Jecko B. Complete FDTD analysis of microwave heating processes in frequency-dependent and temperature-dependent media. *IEEE Trans. MTT*, 1997, 45(1):108–117.

[22] Iskander MF. FDTD simulation of microwave sintering of ceramics in multimode cavities. *IEEE Trans. MTT*, 1994, 42(5):793–800.

[23] Yan L, Huang K, and Liu C *et al.* Study on time scaling factor in the fdtd simulation of microwave heating. *Journal of Sichuan University (Natural Science Edition)*, 2003, 40(2):395–398.

[24] Liu C, Huang K, and Hu Z *et al.* Experimental study and numerical simulation on the microwave nonlinear response due to chemical reaction. *J. Microwaves*, 2000, 16(2):165–171.

[25] James G and John F. Control of thermal runaway – some mathematical insights. *Int. J. Heat Mass Transfer*, 1998, 41:2979–2990.

[26] Groombridge P, Oloyed A, and Siores E. A control system for microwave processing of materials. *J. Manufacturing Science Engineering*, 2000, 122:253–261.

[27] Vriezinga CA. Thermal profiles and thermal runaway in microwave heated slabs. *J. Appl. Phys.*, 1999, 85(7):3774–3779.

[28] Kriegsmann GA and Luke JHC. Microwave joining of two long hollow tubes - an asymptotic theory and numerical simulations. *J. Eng. Math.*, 2001, 39:63–78.

[29] Haala J and Wiesbeck W. Modeling microwave and hybrid heating processes including heat radiation effects. *IEEE Trans. MTT*, 2002, 50(5):1346–1354.

[30] Lee MZC and Marchant TR. Microwave thawing of cylinders. *Applied Mathematical Modelling*, 2004, 28:711–733.

[31] Jerby E, Aktushev O, and Dikhtyar V. Theoretical analysis of the microwave-drill near-field localized heating effect. *J. Appl. Phys.*, 2005, 97:034909–034915.